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Edited by
R. O. GANDY and J. M. E. HYLAND

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TABLE OF CONTENTS

PREFACE vii
CONTENTS viii
INVITED ADDRESSES x

PART ONE: "STUDIES IN LOGIC" SYMPOSIUM

M.D. FRANK, Twenty-five years of "Studies in Logic" 3
H. KEISLER, Hyperfinite model theory 5
G. KREISEL, On the kind of data needed for a theory of proofs 111
W. MAREK, The foundations of mathematics in Poland after World War II 129
H. RASIOWA, A tribute to A. Mostowski 139

PART TWO: HISTORY OF MODERN LOGIC

J. BERG, Bolzano's contribution to logic and philosophy of mathematics 147
R.O. GANDY, The simple theory of types 173
J. VAN HEIJENOORT, Set-theoretic semantics 183
S.J. SURMA, On the work and influence of Stanisław Leśniewski 191
G. TEMPLE, Inference without axiom or paradoxes 221
C. THIEL, Leopold Löwenheim: life, work, and early influence 235

PART THREE: APPLICATIONS OF MODEL THEORY

D.M. CLARK and P. KRAUSS, Relatively homogeneous structures 255
P.C. EKLOF, Applications of logic to the problem of splitting Abelian groups 287
U. FELGNER, Stability and $\aleph_0$-categoricity of nonabelian groups 301
H. SIMMONS, Existentially closed models of basic number theory 325

PART FOUR: RECURSION THEORY

S. FEFERMAN, Inductive schemata and recursively continuous functionals 373
J.E. FENSTAD, Between recursion theory and set theory 393
*R.O. GANDY and J.M.E. HYLAND, Computable and recursively countable functions of higher type 407
J.M.E. HYLAND, Aspects of constructivity in mathematics 439
Y.L. ERSHOV, The model $\mathbb{C}$ of the partial continuous functionals 455

*This paper does not represent an invited address

PART FIVE: COMPLEXITY OF COMPUTATIONS

* K. FLEISCHMANN, B. MAHR and D. SIEFKES, Bounded concatenation as a uniform method for proving lower complexity bounds 471
C.P. SCHNORR, The network complexity and the breadth of Boolean functions 491
R. STATMAN, Complexity of derivations from quantifier-free Horn formulae, mechanical introduction of explicit definitions, and refinement of completeness theorems 505

PART SIX: SET THEORY

T. JECH, Precipitous ideals 521
A.R.D. MATHIAS, The real line and the universe 531
D. PINCUS, Adding dependent choice to the prime ideal theorem 547
R.v.B. RUCKER, The one/many problem in the foundations of set theory 567
J. TRUSS, Sets having calibre $\aleph_1$ 595

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SET-THEORETIC SEMANTICS

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Two fundamental acquisitions of modern logic are the notion of formal system and that of interpretation. These two notions somehow complement each other: when we come to construct a formal language, we abandon the meanings of the symbols in order to deal with their syntactic relations; subsequently, we bring back the interpretation. The birth date of the first notion can be given with complete precision; it is when Frege wrote, in his Begriffsschrift, that the rules he was introducing were rules 'for the use of our signs'. The history of the notion of interpretation is more hazy.

The notion of set-theoretic consequence appears in Bolzano's Wissenschaftslehre (1837), but not for a formal language. Bolzano has no formal notation—he uses ordinary language—but he takes a step toward objectivity by considering what he calls Sätze and Satzformen instead of Urteile and Urteilsformen, these being too psychological. A proposition C follows (ist ableitbar) from the propositions H_1, H_2, ..., H_k, with respect to certain common constituents, if and only if every substitution, taken in a certain class, for these constituents makes C true whenever it makes H_1, H_2, ..., H_k true.

Bolzano's discovery remained isolated. When, after 1870, the development of modern logic began to take some impetus, there appeared two streams that, until 1920, hardly seem to mix their waters. One stream is that of Frege and Russell (to whom we should perhaps adjoin Peano), and the other is that of Peirce, Schröder and Löwenheim. We could say, as an initial approximation, that the first stream is syntactic, while the second is semantic. This is not quite correct, since Frege and Russell have their semantics, these semantics being different from each other and from set-theoretic semantics. Peirce, Schröder and Löwenheim consider domains of individuals, domains on which properties and relations are defined, and most of the time ignore formal proofs; their results, which today we would characterize as model-theoretic results, are obtained by semantic considerations. Schröder has axioms for Boolean algebras, but this does not go beyond the sentential calculus. Most striking in this respect is Löwenheim's system in his famous paper (1915). There Löwenheim deals with first-order logic with identity. But his system is not provided with a notion of proof; there are no axioms and rules. The approach is purely semantic. The notion that Löwenheim handles is that of interpretation, together with the companion notions of validity and satisfiability. We have formulas that are valid, valid in every finite domain, valid in a denumerably infinite domain. Löwenheim does not give an explicit definition of the semantic notions that he uses; he takes them as well-understood; but the novelty and complexity of the results that he is able to obtain leave no doubt that he has a complete grasp of them.

Besides his famous theorem, Löwenheim obtains, by these semantic methods, results about reduction and decision problems, problems in which Frege and Russell

had shown no interest.

To carry out this work Löwenheim introduces a number of technical innovations that will become standard tools of logic, especially in semantic arguments: reduction of a formula to prenex form, a prefiguration of the Skolem functions (to obtain the satisfiability functional form of a formula), a prefiguration of Herbrand expansions (of course, Herbrand found his inspiration in Löwenheim's work), use of an argument that is, in fact, König's lemma.

Löwenheim had learned his logic in the writings of Schröder, who in turn had codified and developed some of the work of Peirce (which branches off in several directions). As early as 1870, Peirce begins to represent binary relations by matrices, finite or infinite, whose elements are ordered pairs of individuals. In 1882 he takes a further step. A relative is represented as a Boolean sum of ordered pairs (taken in a certain domain), each pair being affected by a coefficient, which is 1 or 0 according as the relation obtains between the elements of the pair or not. The mathematical signs Σ and Π thus come to play the role of the existential and universal quantifiers. But these quantifiers are handled by Peirce in a way that is very different from the way Frege handles his quantifiers. There are no axioms about them. Their interpretation and handling are tied up with a specific domain, changeable at will. Moreover, they are generally used only in initial position.

Another feature of the 'syntactic' stream in logic is that Frege and Russell are engaged in a total logical reconstruction of our world. They are interested in starting from a few primitive notions and building a universal system. Therefore their quantifiers range over all objects, that is, the range of these quantifiers is a fixed unique universe, the Universe. This is expressed by Frege in Begriffsschrift: 'All other conditions to be imposed on what may be put in place of a German letter are to be incorporated in the judgment.' (Frege 1879, § 11; van Heijenoort 1967, 24; the sentence is italicized in the original.) This means that, when we deal with a special class of objects, we use the method known as relativization of quantifiers. 'For every natural number ...' becomes translated as $\forall x(Nx \supset \ldots)$, where $\forall x$ ranges over all objects. This is what Frege means by incorporating the condition in the judgment. In Russell we have a stratified universe, but each stratum is a fixed domain. This universality of the Frege-Russell logic is discussed in van Heijenoort 1967a.

With a fixed universe the semantic notions of validity and satisfiability lose their applicability, while with a variable universe they come to the fore. As early as 1885, Peirce gives the correct definition of validity for the sentential calculus: 'To find whether a formula is necessarily true substitute f and v for the letters and see whether it can be supposed false by any such assignment of values.' (Moreover, for testing validity Peirce suggests a method that foreshadows refutation trees.) If Frege and Russell come to consider the formula, say, $\forall x(Fx \supset Fx)$, it is only as a temporary equivalent of $\forall F \forall x(Fx \supset Fx)$, and for this latter formula the notions of validity and satisfiability come to coincide and collapse into the notion of truth.

Closely connected with these considerations is, of course, the question of first-order logic as a separate system. Embarked in their grandiose logical reconstruction, Frege and Russell do not hesitate to go beyond first-order logic. And they have to, since the notion of the ancestral is necessary for their definition of natural number. When Frege passes from first-order logic to a higher-order logic (1879, § 11; van Heijenoort 1967, 24), there is hardly a ripple. Not bound by large-scale requirements, Peirce, Schröder and Löwenheim can more distinctly feel the ground under their feet, and then, of course, the difference in complexity between first-order and higher-order logic is immediately apparent. The birth and development of model theory would hardly have been conceivable without the separate consideration of first-order logic.

Hilbert's position is somewhat between that of Frege-Russell and that of Peirce-Schröder-Löwenheim. Like the former, he works with axioms and rules. With his mathematician's instinct, however, he is inclined to consider quantifiers ranging not over 'everything', but rather over well-defined collections of objects. He also feels that the jump from first-order to second-order logic marks an important change in complexity; and, though second-order logic may be indispensable at a certain stage, it is important to see what can be done in first-order logic. So, when the fusion of the two currents took place in the Twenties, Hilbert was one of its agents. The notion of a formal deductive system supplemented by an interpretation appears in Hilbert's logical writings, and in Hilbert and Ackermann 1928 the problem of the completeness of first-order logic is posed in exactly the terms in which it will be solved in Gödel 1930.

Another agent was Skolem. He knew, of course, Löwenheim's work, which he had amended and generalized. But he had also read Principia. He had at hand the various arguments that constitute the proof of completeness, but the very import of the proof seems to have remained hazy for him. Rather than using axioms and rules, he kept looking for a proof procedure, perhaps similar to the one that Herbrand was going to propose. Skolem's work raises many delicate questions of interpretation, which I do not intend to discuss here. I will simply add that his work did not have, at the time, an impact comparable to that of Hilbert's; Skolem's papers, in the Twenties, were hardly read by the logicians that were then shaping logic.

In 1930, when the proof of the completeness of first-order logic with respect to set-theoretic semantics was presented, there were two other semantics, that of Brouwer and that of Herbrand.

Brouwer was certainly concerned with meaning. In fact, a great part of his polemics against Hilbert turned around the notion of Inhalt. But neither he nor Heyting thought of systematizing the notion. I have no intention of discussing intuitionistic semantics here.

Herbrand, with his (feigned ?) acceptance of the Hilbert program, rejected set-theoretic considerations that were not decidable. Instead of the set-theoretic notion of satisfaction, he uses a notion based on an infinitely proceeding sequence of finite domains, existential quantifiers having a range different from that of the universal quantifiers. No doubt, Herbrand had the notion of set-theoretic interpretation present in the back of his mind all the time. Only thus can his errors be explained. The semantic argument for Herbrand's theorem is incomparably simpler than the syntactic argument, so that, being convinced by the first, Herbrand did not bother to check carefully the second (in the same way that, when we have a simple geometric argument, we are prone to make a mistake in the complicated calculations of the parallel analytic argument, simply out of boredom).

Compared to the glamorous axiom systems, which, with so few axioms and rules, claimed to encompass a large tract, if not all, of mathematical practice, semantic considerations seemed to be a homely breed. Gentzen, in 1935, still prefers to establish the constructive equivalence of his calculus of sequents with some existing system, rather than to prove directly the completeness of the new system (he may have thought that merely giving such a proof was a bit like cheating). It is Tarski's codification of set-theoretic semantics, in the mid-Thirties, that finally gave an honorable status to semantic considerations and launched them toward model theory.

Model theory has bloomed into an extensive science. But this new discipline does not seem to care much about what was the original object of semantics, namely meaning. It has become an abstract mathematical science, whose philosophical implications are scant. More precisely, model theory does not seem to have anything to contribute to a theory of meaning, if such a theory has to deal with

ordinary language. The only philosophical implications of model theory are at a highly technical level, where results in set theory may affect our views of what is mathematics.

Recently, attempts have multiplied to construct a semantics of ordinary language on a precise basis. In the last hundred years the views of the philosophical community about natural language seem to have gone through a three-stage dialectical process. Frege and Russell wanted to replace natural language with a formal language, because of the imprecision, vagueness, haphazard complexity of the former. The natural language is not considered to be a worthy tool of philosophical investigation, not even a worthy object of study. We may get suggestions from it, but it is no more than a ladder that we kick back, once we have been able to reach the formal language. As a reaction against this point of view, Wittgenstein, in his second period, wanted to leave natural language as it is, not replace it by anything else. The niceties and convolutions of natural language express subtle differences in thought, which we must examine with care and patience. Today, we have entered a third phase. Philosophers have been reminded by linguists that natural language, in its syntactic aspects, can be an object of exact study, a study that, with its generalizations, is reminiscent of that of artificial languages. Then, more recently, this exact study attempts to encompass also the semantic aspects of natural language. I do not intend to survey here the whole field, but to touch upon a number of points.

In Tarski's well-known paper on set-theoretic semantics, the only philosophical remarks deal with the Aristotelian realism of his definition of truth. But the semantics rests on specific ontological assumptions that have hardly been made explicit: we have bare individuals; these individuals have no inner structure, they are mere pegs. That is why we often speak of 'elements', even sometimes of 'points'. We assume that we have a collection of such individuals, distinguishable by its cardinality. Properties and relations come subsequently and do not contribute to the identification of the individuals. On the contrary, the individuals have to be individuated so that properties and relations can be introduced. How are the individuals individuated ? This is left out of the account. What we have here is a highly abstract scheme. Can this dessicated ontology encompass everything we want to talk about ? Is it applicable to material objects, to events, to acts ?

Just to question the universal applicability of set-theoretic semantics I would like to raise here the question of mass terms.

These terms, like water, gold, dirt, are opposed in their syntax to the so-called count terms, like men, tables, chairs. With them we use the question How much ?, instead of How many ? They obey peculiar syntactic rules. It is to be noted that in different languages, English and French for instance, the classes of mass terms are quite similar (that is, correspond under translation), while the syntactic rules that allow us to characterize these classes are quite different (that is, do not correspond under translation). This fact seems to give weight to the assumption that the class of mass terms is a stable and significant linguistic object.

Mass terms can, in a sentence, be used before the copula (Iron is a metal) or after (This is gold). They can be used with demonstratives (This water is dirty), possessives (My coffee is cold) or the definite article (The wine that you gave me was sour), to yield what I would call quasi-individual terms. They can also be used with so-called container words (three buckets of water, two pounds of sugar) in a variety of ways: sometimes the units are individualized, sometimes not. There is here a rich field for linguistic investigations.

Concrete mass terms, like those we have mentioned so far, are important because of the linguistic problems they raise. But their importance is increased by the fact that abstract terms, like courage, wisdom, and so on, behave, linguistically, like them.

Brave attempts have been made to deal with mass terms by means of set-theoretic semantics. Some have taken them as denoting (scattered) individuals (Quine), some as referring to properties (Strawson). These attempts can be pushed up to a certain point, then end up by doing violence to the linguistic facts. The most natural way to deal with mass terms seems to be to provide them with their own semantics. As we have a universal domain of individuals for count terms, we now consider a universal lump for mass terms, a lump out of which we take slices.

The two semantics may have to be used concurrently. First, because a sentence may contain both count terms and mass terms, as The man drank some water. Second, because the quasi-individual terms that I just mentioned, obtained from mass terms by a variety of linguistic means, require individuals for their interpretation.

There is also, with mass terms, the problem of mixtures, both on the syntactic and on the semantic plane.

Since quantifiers cannot be interpreted except in a discrete domain, the proper treatment of mass terms requires a variable-free system. But several such systems are at hand today, and there does not seem to be any difficulty in adapting one of them to the semantics of the universal lump. One can only wonder why all that has not been done before. It is ironic that Tarski's famous example, 'Snow is white', involves a mass term, and Tarski's semantics would have met with difficulties if sentences containing other uses of 'snow' had been considered. My purpose in raising here the problem of the semantics of mass terms has been to try to perceive the limits, and limitations, of set-theoretic semantics.

An ambitious attempt to apply to natural language the methods and procedures acquired in the study of formal systems was made by Richard Montague. His system went through several phases, and it is not my intention to discuss it here in its entirety. I would like to discuss a point on which it differs markedly from standard set-theoretic semantics, namely, on the notion of individual, and I take Montague's doctrine as presented by Barbara Partee after his death (Partee 1975).

For Montague an individual is the collection of all properties that are true of the individual, in our sense. The semantic interpretation of John is the collection of all properties that hold of John. This conception is certainly different from our set-theoretic semantics and offers an alternative semantics, which, moreover, has a long philosophical tradition (Leibniz). With this semantics, Montague is able to assign a semantic denotation not only to names (individual constants and definite descriptions), but also to expressions like every man or some man. Or rather, these expressions become, in a way, names, on the same plane as John. The denotation of every man is the collection of all properties that hold of every man, that is, it is the intersection of the collections assigned to John, Paul, and so on. The denotation of some man is the collection of properties that hold of at least one man, that is, the union of the collections assigned to John, Paul, and so on.

Barbara Partee gives a list of contexts in which the substitution of expressions like every man or some man for John leads from a meaningful sequence of English words to another. There are, however, limits to this parallelism, and these limits are so fundamental, linguistically, that the unification becomes doubtful. I now list three such limits.

(1) Negation. The negation of John came is John did not come. The negation of Every man came is not Every man did not come, but Not every man came. And so on. Examples could be multiplied. Negation is such an important linguistic and logical notion that these differences cannot be considered to be innocuous and chance peculiarities of the language. They are rooted in its functioning.

(2) Scope. In John loves Mary there is no problem of scope. But, with a and every, the problem of scope arises, as in A woman loves every man. Here again

examples could be multiplied. And here again we are confronting something basic in the functioning of the language. The problems of scope of quantifiers in English are of a degree of complexity that is no less than in quantification theory, as one can convince oneself by a few examples. Hence we cannot hope to get by through the use of a semantics that does not respect this complexity.

(3) Inference. From John gave exactly one book to Mary and none to anybody else, we can infer John gave exactly one book. But we cannot infer this last sentence from John gave exactly one book to every student and none to anybody else. The substitutions mentioned above disturb inferences. And here again the same remarks can be made as for (1) and (2).

This list is certainly not exhaustive. Other limits, on the linguistic plane, can be found to the unification proposed by Montague. On the philosophical plane, a number of objections present themselves. Without entering here into a full discussion of the problem, let me mention that Montague's conception, at least as presented so far, would make all statements analytic. If the denotation of John is a collection of properties, then John came simply states that one specific property is in the collection in which it was put.

A great deal of the present activity in the field of the semantics of natural languages is devoted to the study of the relations obtaining among terms, like between fox and vixen, in the hope that, if we could codify these relations, we would be able to read off the logical entailments that obtain among sentences of a natural language like English. The entailment of

(1) No man lives forever

by

(2) All men are mortal

should emerge from the proper semantic analysis of mortal, to live and forever. And we should be able, progressively, to deal with more and more complex examples. What is attempted, or rather contemplated, is the construction of a dictionnaire raisonné. But this an old enterprise. Descartes was already asking for an enumeration of all the idées simples and their hierarchization. Such an enterprise works—up to a certain point. There are in English perhaps a few thousand words that lend themselves pretty nicely to such a work of regimentation. But soon doubts and difficulties multiply. Try to unravel a skein of yarn; you will pull a good length of it, but then you will have a worse mess than before. One begins to grope for guidance and would like to have rules and principles. But these are notoriously lacking. The very idea of idée simple is not simple. Where is the line to be drawn between a dictionary and an encyclopaedia ?

I do not want to claim that such work cannot be done. But, before it can be done (perhaps with the help of computers), certain fundamental questions have to be answered, questions that have not yet been formulated properly.

A pervading assumption, implicit or explicit, underlying the construction of a rational dictionary is that we have to proceed from the simple to the complex. And how could it be otherwise ? But the realities of a natural language force us to temper such a view. We do not simply proceed by addition, composition, conjunction. Consider the sentences He squeezed a lemon and He squeezed my hand. Don't we have here a kind of feedback effect, through which the meaning of the complement alters the meaning of the verb ? Consider red in red wine, red face and red hair. The meaning of the noun affects the meaning of the adjective. We do not simply have conjunction. When words are put together, they interact on one another, and there are effects that the inductive mode of definition, used for artificial languages, is not quite able to catch. Should we then consider words only in expressions ? Should a dictionary be made up of examples, rather than of

attempted definitions ?

Consider now an example that has been much discussed by linguists. The correct French translation of

(3) He swam across the river

is

(4) Il traversa la rivière à la nage.

The word by-word translation of (3), which is

(5) Il nagea à travers la rivière,

has no clear meaning in French. We translate to swim across by traverser à la nage. We have here something reminiscent of idioms. When we translate I am hungry by J'ai faim, we do not use a word-by-word transformation either, and linguists are prone to keep idioms out of their theoretical reconstructions (see, for instance, Fodor 1972, page 50). However, he who has some practice in translating knows that the idiomatic aspect that reveals itself when we pass from (3) to (4) is no exception. One would be tempted to say that it is rather the rule. Once a translator has paired off a word of the target language with each word of the original text (perhaps with the help of a dictionary), he rereads the whole thing, catches the meaning, and writes down the translation. There is no upper bound upon the length of the rearrangement. We cannot isolate a definite class of idioms. The idiomatic effect pervades the whole language.

But, if we do not have a rational dictionary, if we are not even sure of how such a dictionary can be written, does it mean that we cannot deal with entailments in a natural language, like that of (2) by (1) ? Of course, we can. Logic, even today, allows us to deal with the semantics of a natural language locally. That is, we translate the sentences into the language of quantification theory, and then use standard set-theoretic semantics. This is what we do when, in a beginners' course in logic, we give examples. It is a rough job. Many things get bulldozed. Given a sentence that we consider to be logically atomic, we pick one, two or three parts in it, which, we decide, work as individual terms, and we push everything else into the predicate. Adverbs suffer, and many other things. But, nevertheless, within the limits of what we intend to do, it works. If somebody claims that there is a relation of entailment among certain English sentences, we can check, by translating these sentences into quantification theory, whether the entailment obtains or not (check, that is, within the limitations imposed by the decision problem; but it is rare that an example suggested by natural language is not in a decidable fragment). If one and the same sentence occurs in two different sets of sentences, we may have to translate it differently if we want to reveal the entailment corresponding to each set, but in each case the operation is possible.

There is nothing strange and mysterious about checking that (2) entails (1). We simply adjoin the side assumption

(6) For every x, x is mortal if and only if x does not live forever.

If our dictionnaire raisonné had been completed, we would read (6) off the dictionary. As long as the dictionary is not at hand, we adjoin one or several side assumptions. If we understand the language, this presents no difficulty.

We could compare the natural language to a non-Euclidean space, for which there is, at each point, a tangent Euclidean space. In a neighborhood, the tangent space can take the place of the given space. If we go too far, the distortion becomes too great.

The problem is to pass from the local treatment of the semantics of a natural language to a global one. One feels that it is possible and that it should be done. Why ? Because the handling of words is an exact and precise activity. Whoever writes with some care knows that replacing one word by another, apparently close, may change the meaning of a sentence appreciably; even a comma, added or taken away, as its effect. The syntactic codification of such facts cannot be the full answer. Unless God-given, syntactic rules must be grounded somewhere. And where, if not in meaning ? But between the exact working of a natural language, with the convolutions and intricacies of its meanings, and the logical skeleton representable in quantification theory there is a large gap, in which we are still groping.

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